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Phenomenological Aspects of Viscoelastic Crack Propagation

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Synopsis

The strain rate dependence on dynamic crack propagation in viscoelastic solids was experimentally verified for PMMA by the velocity gauge method and fracture surface observation. It was found that the dependence was mainly due to the viscous term effect, and this was supported by the observation of deviation of the stress–strain curve from an elastic case. The elastic brittle crack propagation can be realized only in the high-strain rate case for PMMA.

INTRODUCTION

So far, viscoeleastic materials with high modulus, such as poly(methyl methacrylate) (PMMA), have been treated as brittle solids so that the elastic brittle fracture phenomena which are strain rate-independent, such as dynamic crack propagation, have been expected. However, the present authors believe the effects of viscoelasticity may be important at low applied strain rates. Such strain rate dependence on dynamic crack propagation in a viscoelastic solid such as PMMA is little investigated. In this exploratory experimental study using PMMA, the phenomenologic aspects concerning the strain rate dependence on dynamic crack propagation in viscoelastic solids are discussed.

EXPERIMENTAL

Measurement of Crack Velocity

The measurement of crack propagation velocity through a specimen was done by the velocity gauge method.[1] Velocity gauges consist of a series of conducting wires, du Pont No. 4817 conductive silver coating material, placed at certain intervals on the projected path of the crack and perpendicular to the direction of crack propagation, as shown in Figure 1. These wires form one leg of a bridge, shown in Figure 2, which is connected to a Synchroscope DS-5305B, made by Iwatsu Electric Co., Ltd., Japan. The

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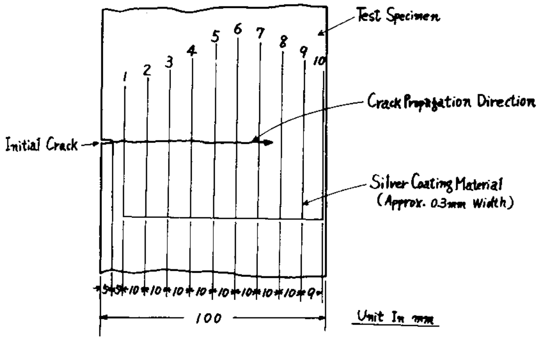


Fig. 1. Velocity gauge arrangement.

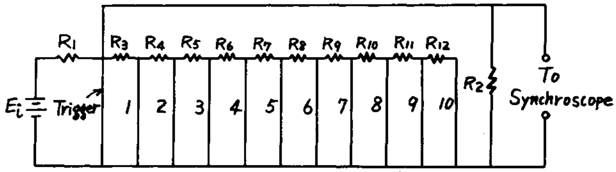


Fig. 2. Electronic circuit.

times at which these wires break owing to the propagating crack are obtained from the trace on the Synchroscope. Thus, the average crack propagation velocity between wires can be obtained.

Tensile Testers

Two Instron-type tensile testers were used to give a constant strain rate ranging from $\dot{\epsilon} = 2.6 \times 10^{-7}$/sec to $\dot{\epsilon} = 48$/sec, as shown in Table I. For $\dot{\epsilon} = 48$/sec, the UTM-5 tensile tester, specially designed for higher strain rate tension loading, was used. In this case, an initial inherent transient strain rate range, that is, an inherent transient cross-head speed range, is inevitable, since this UTM-5 tester is originally designed to fit elastomers showing large deformations. Therefore, a "slack grip" was installed to eliminate the transient effects, of which details were described in the earlier paper,[2]. The breaking load was measured by a load cell. For $\dot{\epsilon} = 2.6 \times 10^{-2}$/sec, $\dot{\epsilon} = 2.6 \times 10^{-5}$/sec, and $\dot{\epsilon} = 2.6 \times 10^{-7}$/sec, the conventional-

TABLE I
Tensile Testers

Tensile tester	Strain rate $\dot{\epsilon}$, sec^{-1}	Make
UTM-1	2.6×10^{-7}	Toyo-Baldwin, Japan
	2.6×10^{-5}	
	2.6×10^{-2}	
UTM-5[a]	48	

[a] A "slack grip" was attached.

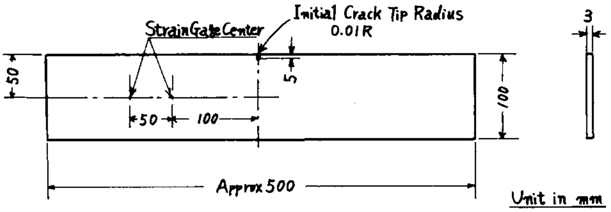


Fig. 3. Test specimen for low strain rate.

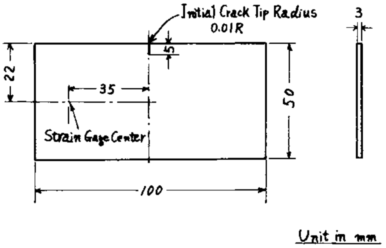


Fig. 4. Test specimen for high strain rate.

type tensile tester UTM-1 was used. No "slack grip" was used in these cases. Both UTM tensile testers are of Toyo-Baldwin (Japan) make.

Test Specimens

Figure 3 shows for both quasi-static and slow strain rates, say, from $\dot{\epsilon} = 2.6 \times 10^{-7}$/sec to $\dot{\epsilon} = 2.6 \times 10^{-2}$/sec, and also shown in Figure 4 is for a high strain rate of $\dot{\epsilon} = 48$/sec. These PMMA specimens were made from Sumipex virgin sheet manufactured by Sumitomo Chemical Co., Ltd. Japan. The initial crack length c_0 was 5 mm, and the crack tip radius r was 0.01 mm. One or two strain gauges, KFC-2-500-C1-11, manufactured by Kyowa Electronic Instruments Co., Ltd. Japan, were placed on the specimen as shown in the figures, and were measured on the Synchroscope DS-5305B. The velocity gauge installation was applied to all the specimens as mentioned above, however, the number of conducting wires were reduced by half for those shown in Figure 4 because of shorter width.

EXPERIMENTAL RESULTS

The experimental results of crack propagation velocity in PMMA by the velocity gauge method are shown in Figures 5 to 8, where c = crack length and $\dot{c}$ = crack velocity. In Figure 5, the strain rate is very low, $\dot{\epsilon} = 2.6 \times 10^{-7}$/sec, and the asymptotic value is near 400 m/sec. In Figures 6 and 7, for $\dot{\epsilon} = 2.6 \times 10^{-5}$/sec and $\dot{\epsilon} = 2.6 \times 10^{-2}$/sec, respectively, the asymptotic values are being raised; and finally in Figure 8, for $\dot{\epsilon} = 48$/sec, the estimated terminal crack velocity becomes about 680 m/sec, which is almost the theoretical crack propagation velocity in brittle elastic solids, as discussed later. An arrangement of these data, shown in Figure 9, denotes a distinct strain rate-dependent crack propagation velocity profile in PMMA.

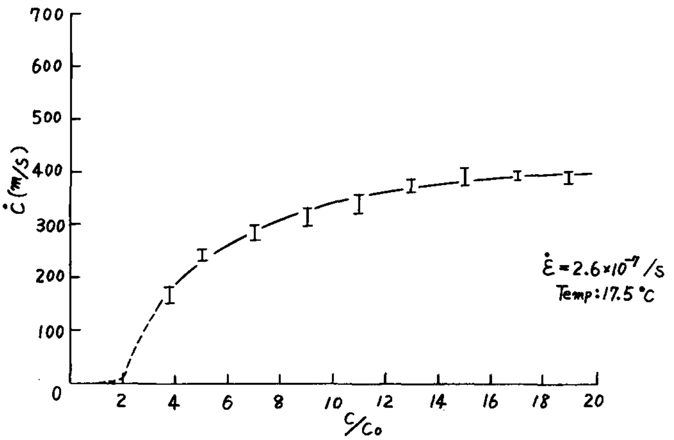


Fig. 5. Crack propagation velocity profile ($\dot{\epsilon}$ = 2.6 × 10^{-7}/sec).

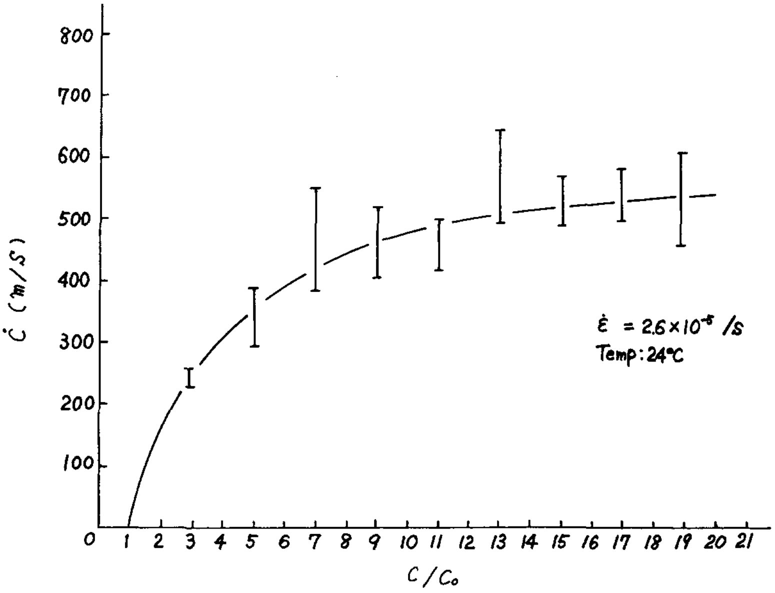


Fig. 6. Crack propagation velocity profile ($\dot{\epsilon}$ = 2.6 × 10^{-5}/sec).

Another experimental result, shown in Figure 10, also supports the strain rate-dependent crack propagation in PMMA. Figure 10 shows the topologic change of fracture surfaces taken in micrographs with a magnification of 177×, of which details are presented separately in parts A to N. These micrographs display a gradual increase of roughness in the fracture surface with increase in crack velocity, as was already known, e.g., see Cotterell.[3] In Figure 10, comparing the upper curve ($\dot{\epsilon}$ = 48/sec) with the lower one ($\dot{\epsilon}$ = 2.6 × 10^{-7}/sec), it can be observed that the lower strain rate case displays a smoother fracture surface in contrast to the higher strain rate. The rough fracture surface indicates a higher crack velocity for higher strain rates.

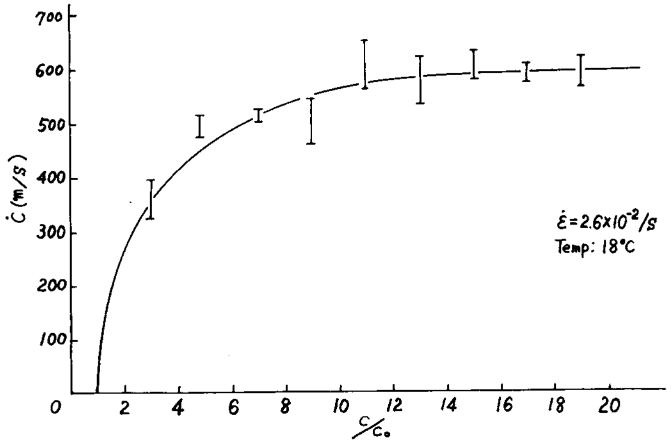


Fig. 7. Crack propagation velocity profile ($\dot{\epsilon} = 2.6 \times 10^{-2}$/sec).

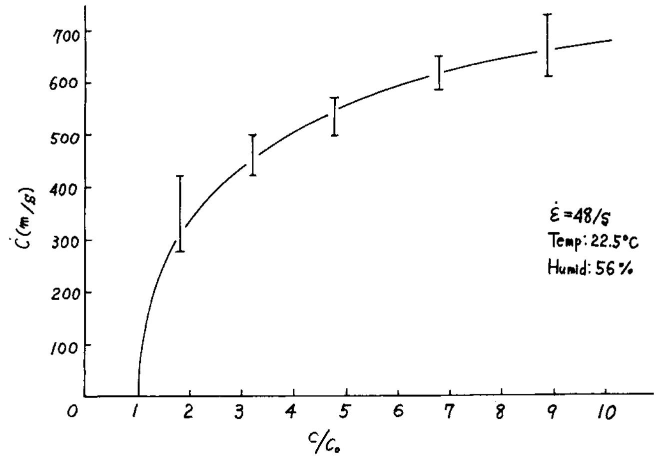


Fig. 8. Crack propagation velocity profile ($\dot{\epsilon} = 48$/sec).

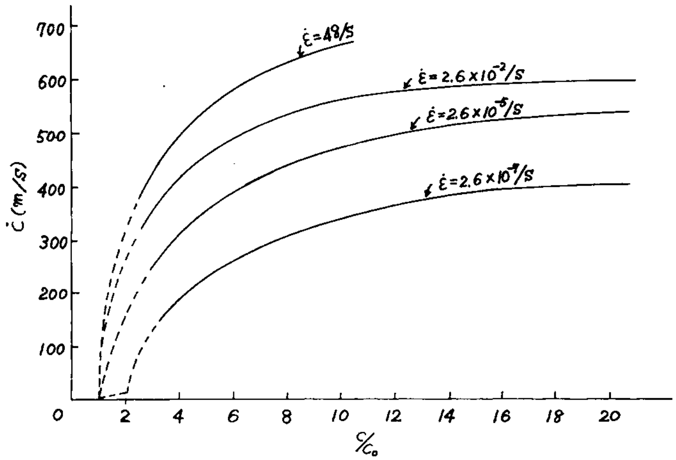


Fig. 9. Strain rate-dependent crack propagation velocity profile.

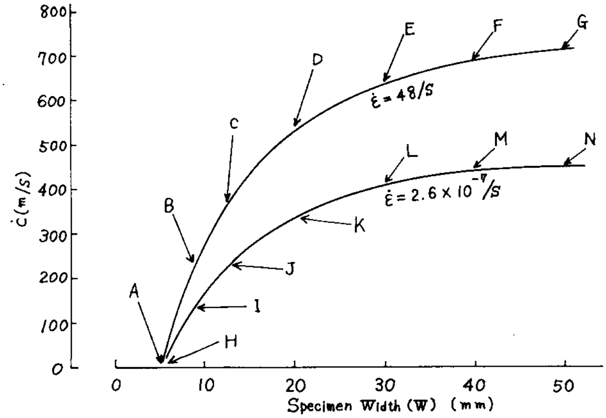
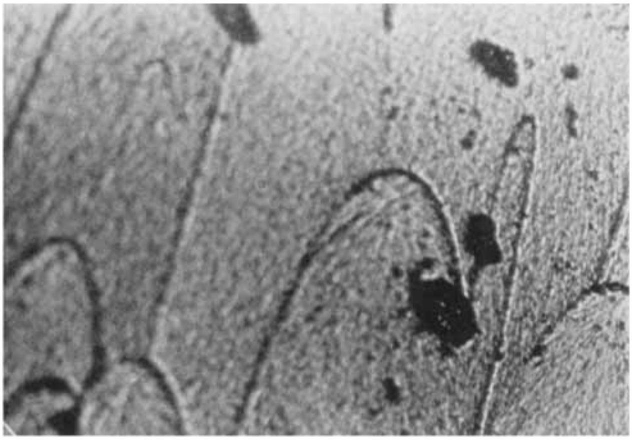
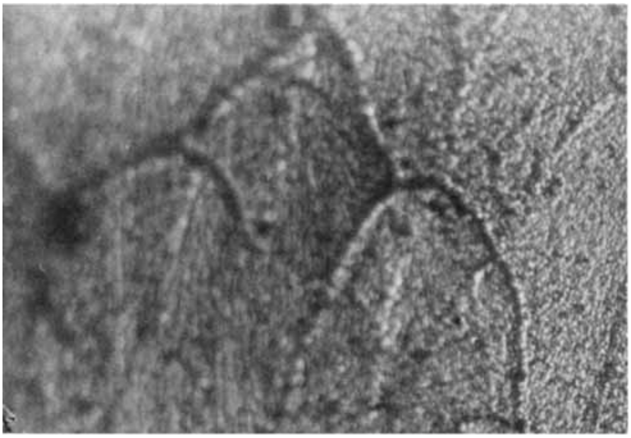


Fig. 10. Positions of micrographs taken on crack propagation velocity profile (see Figs. A to N).

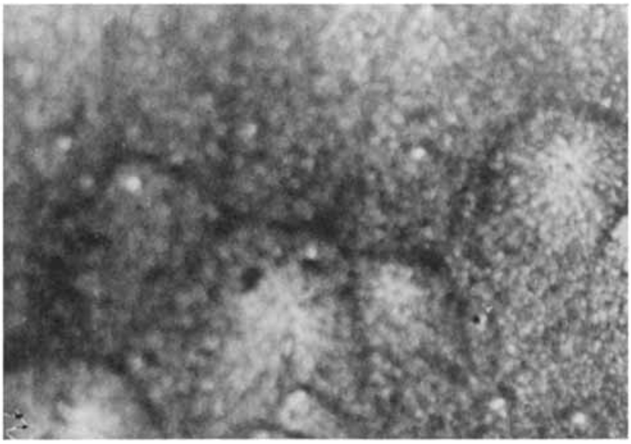


(A)

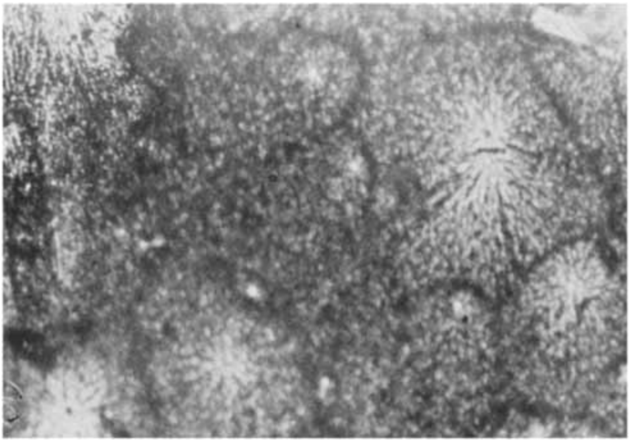


(B)

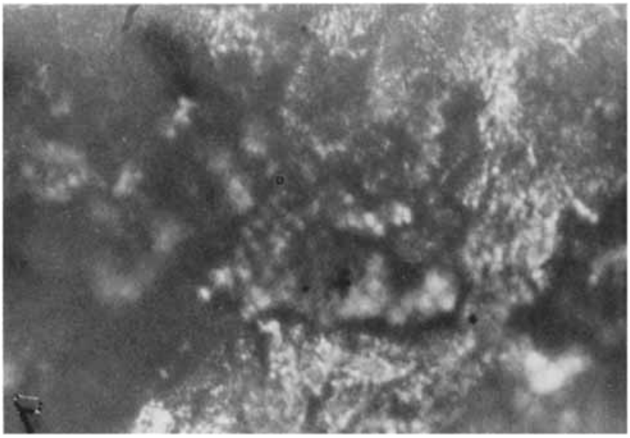
Fig. 10 (*continued*)



(C)

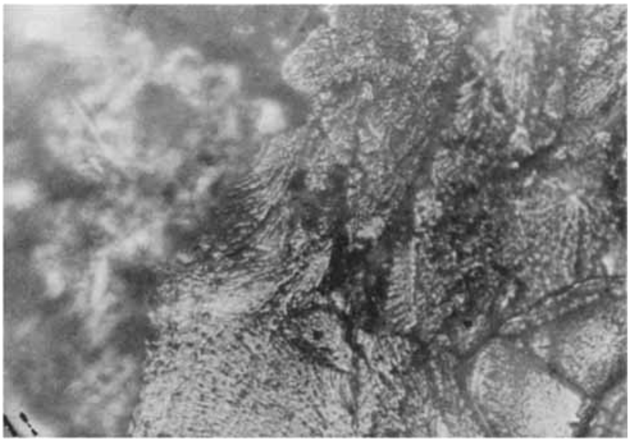


(D)

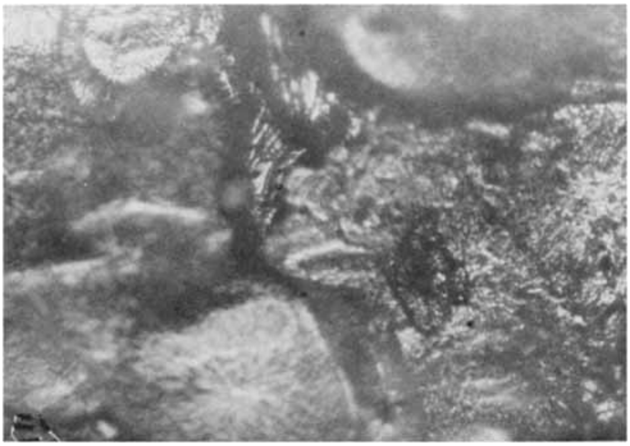


(E)

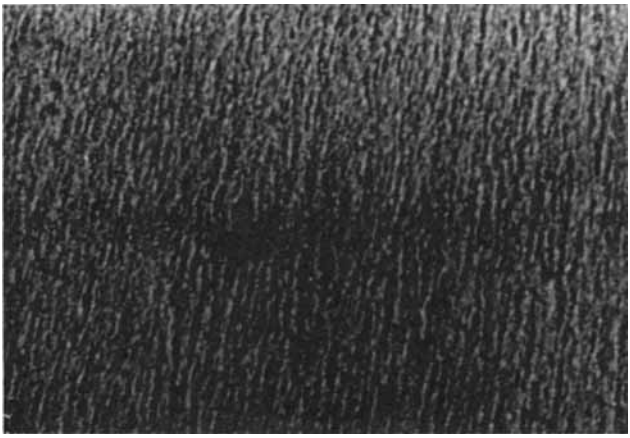
Fig. 10 (*continued*)



(F)

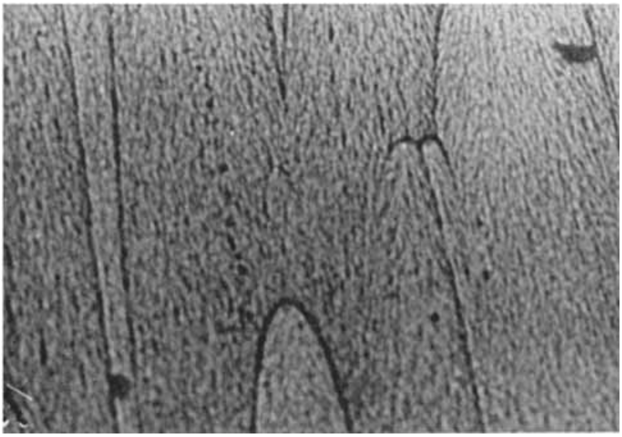


(G)

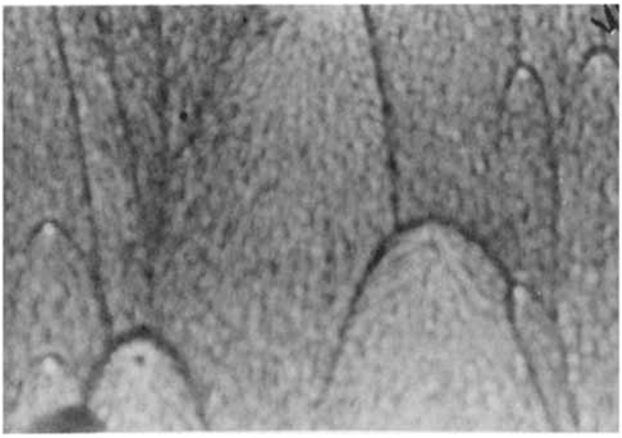


(H)

Fig. 10 (*continued*)



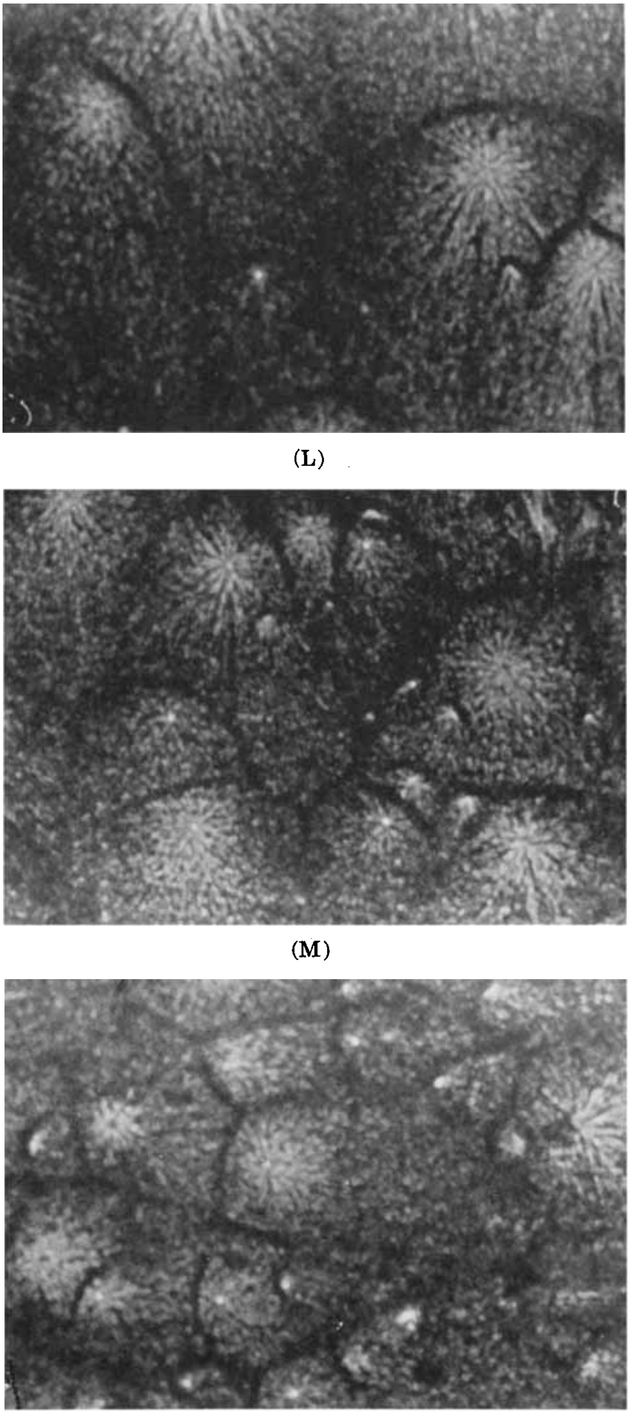
(I)



(J)

(K)

Fig. 10 (*continued*)



(L)

(M)

(N)

Fig. 10.

THEORETICAL INTERPRETATION AND ASSOCIATED EXPERIMENTAL VERIFICATION

A linear viscoelastic solid such as PMMA may be expressed in terms of a four-parameter model, as shown in Figure 11, in which the elastic moduli E_1 and E_2 and the viscosity coefficients η_1 and η_2 can be determined by a creep test or a relaxation test. In the present report, the creep test was used, and $E_1 = 418$ kg/mm^2, $E_2 = 1640$ kg/mm^2, $\eta_1 = 5.84 \times 10^{-6}$ kg sec/mm^2, and $\eta_2 = 1.18 \times 10^{-4}$ kg sec/mm^2 were obtained. For high strain rate loading, only E_1 will be expected to operate instantaneously as soon as a load is applied, since the viscosity terms show the delayed behavior in response to such dynamic loading.

If only E_1 is in operation, the model should exhibit elastic behavior, thus showing the elastic brittle fracture which obeys Berry's formula for dynamic crack propagation.[4] Berry's formula is expressed as

$$\dot{c} = 0.38 \sqrt{\frac{E}{\rho}} \sqrt{1 - \left(\frac{c_0}{c}\right)^2 - 2\left(\frac{\sigma_g}{\sigma_f}\right)^2 \frac{c_0}{c}\left(1 - \frac{c_0}{c}\right)} \tag{1}$$

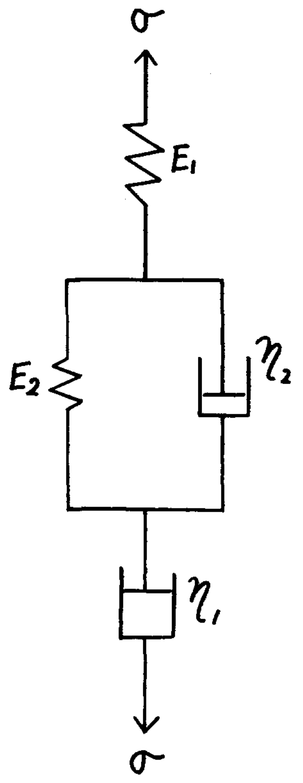


Fig. 11. Four-parameter model.

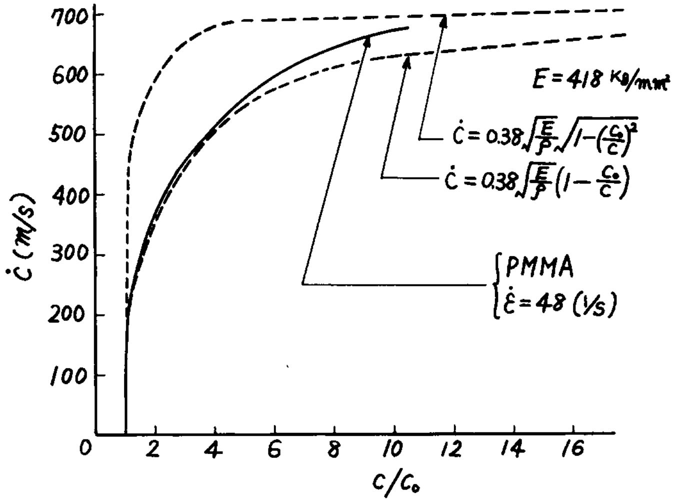


Fig. 12. Crack propagation velocity profile at $\dot{\epsilon}$ = 48/sec compared with Berry's formula.

where $\dot{c}$ = a crack velocity; c = crack length; E = Young's modulus; ρ = the density; c_0 = initial crack length; $\sigma_g = \sqrt{2\gamma E/(\pi c_0)}$ = Griffith's fracture stress at $c = c_0$, where γ = the surface energy; and σ_f = fracture stress.

Equation (1) has both an upper and a lower boundary, i.e., for the upper boundary, $\sigma_f = \infty$,

$$\dot{c} = 0.38 \sqrt{\frac{E}{\rho}} \sqrt{1 - \left(\frac{c_0}{c}\right)^2} \tag{2}$$

and for the lower boundary, $\sigma_f = \sigma_g$,

$$\dot{c} = 0.38 \sqrt{\frac{E}{\rho}} \left(1 - \frac{c_0}{c}\right). \tag{3}$$

Putting $E = E_1 = 418$ kg/mm^2 in eqs. (2) and (3), and comparing with the experimental data obtained for $\dot{\epsilon}$ = 48/sec shown in Figure 8, good agreement is seen with Berry's formula as shown in Figure 12.

Since the dynamic crack propagation velocity is governed by the energy criterion, it is not unreasonable that the dissipation energy due to the viscous flow essential to viscoelastic solids like PMMA is likely to exert a predominant role in the above-mentioned experimental results. In the present test results, for $\dot{\epsilon}$ = 48/sec, the crack velocity agrees well with that of the elastic case as shown in Figure 12. Therefore, it may be concluded that there is insufficient time for viscous deformation under such high strain rate, and only the eleastic component E_1 operates, as can be easily understood from Figure 11.

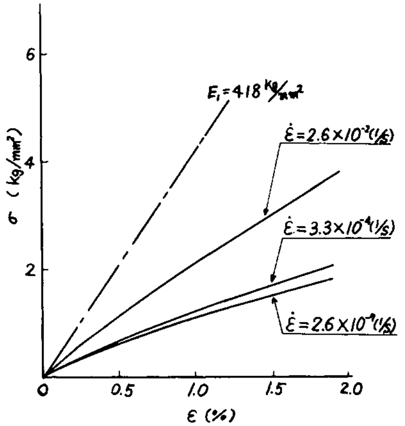


Fig. 13. Stress σ vs. strain ϵ at three different strain rates.

However, for the low strain rates, there is always enough time for the elastic strain to relax, and viscous flow emerges. If such viscous flow existed, first, the stress σ-versus-strain ϵ curve would appreciably deviate from a straight line of purely elastic gradient due to the creep phenomenon. By using a UTM-1 tensile tester, the stress σ-versus-strain ϵ curves for three different strain rates were obtained, as shown in Figure 13, which exhibits more deviation from the initial elastic gradient E_1 with decrease in strain rates. Thus, the existence of viscous flow due to the viscous term in PMMA in the low strain rates is experimentally verified. Therefore, it might be concluded that the strain rate dependence of the dynamic crack propagation in PMMA is mainly due to the viscous term, which appears as the strain rate becomes lower, and that the elastic brittle crack propagation can only be realized at very high strain rates where only the elastic term can be in operation.

CONCLUSIONS

The strain rate dependence on viscoelastic crack propagation was experimentally verified for PMMA by using the velocity gauge method and observations of the fracture surface. The viscous term becomes important as the strain rate decreases. The experimental investigation of how the strain rate causes deviation of the stress–strain curve from the perfectly elastic case is consistent with the effect of strain rate on crack propagation. The elastic brittle crack propagation after Berry's formula was only realized in the high strain rate case for such a viscoelastic solid as PMMA.

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References

1. H. Liebowitz, Ed., *Fracture*, Vol. II, Academic Press, New York, 1968, p. 545.
2. A. Kobayashi and N. Ohtani, *J. Appl. Polym. Sci.*, **15**, 975 (1971).
3. B. Cotterell, *Appl. Mater. Res.*, 227 (Oct. 1965).
4. J. P. Berry, *J. Mech. Phys. Solids*, **8**, 194 (1960).

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